Tentamen Dynamics of Galaxies

January 27, 2009, 10:00–13:00 uur

Write on your first sheet **your name and student-number** *and on all subsequent sheets at least your name.*

N.B. *Each part of the questions below has equal weight in the final grade. When a question us divided in parts and if only some of these are answered correctly the grade for that question is determined in proportion to what fraction of the answers is correct. Not all parts have equal weight: Part* **(1.)** *has weight 40%,* **(2.)** *30% and* **(3.)** *also 30%.* You have in total three hours so divide up your time accordingly.

(1.) Explain and discuss the following concepts. Formulate that as if I were someone familiar with astronomy but without having followed a course on galactic dynamics; try to convey what it is and how it can help us understanding galaxies. Remember that I should be able to judge that you have understand what the concept is. Where appropriate write down the relevant equation(s) (you don't have to memorize them, so select from the list attached).

- Epicyclic frequency
- Asymmetric drift
- Toomre Q
- Inner Lindblad resonance
- Two-body relaxation
- Violent relaxation
- Dynamical friction
- Isothermal sheet
- $V_{\rm m}/\bar{\sigma} - \epsilon$ relation
- Stäckel potentials

(2.) The collisionless Boltzmann equation can be written as the equations of motion for a stellar orbit in a given potential. When these are solved we find integration constants that are called integrals of motion.
What are these and why are they named integrals of motion? Can you give examples of integrals of motions?
What is an isolating integral? How many are there in a spherically symmetric potential?
How many isolating integrals are there in general in an axisymmetric potential as in a disk of a galaxy and what are they physically? What if the motion in the vertical direction is small?
What is a "reduced potential"? What is the "zero velocity curve"? What is the "third integral problem"?
What is the Jeans theorem? And what has Lynden-Bell added to this?

(3.) Write a short review on the subject of "maximum disk hypothesis". Mention in any case how one determines the contributions of various components in a disk galaxy (and what are these possible components?). What is the general behavior of the rotation curve of a self-gravitating exponential disk and what is the free parameter in fitting rotation cureves? What is an isothermal sphere and why can it be used here as a description for dark matter halos? How could one in principle prove or disprove the hypothesis and what do you think, on the basis of what you heard in the lectures, is balance opf the evidence for or against the hypothesis? (You do *not* have to agree with me!)

These pages list many equations used in galactic dynamics in a random order and without any explanation. These you can use in your answers, but then you will need to explain what the properties are. They only serve to help you use them without having to learn them by heart.

$$\frac{\partial}{\partial R}(\nu\langle V_{\rm R}^2\rangle) + \frac{\nu}{R}\{\langle V_{\rm R}^2\rangle - V_{\rm t}^2 - \langle(V_\theta - V_{\rm t})^2\rangle\} + \frac{\partial}{\partial z}(\nu\langle V_{\rm R}V_{\rm z}\rangle) = \nu K_{\rm R}$$

$$\kappa = 2\{-B(A-B)\}^{1/2} \;\; ; \;\; \Omega_{\rm p} = \Omega_{\rm rot}(R) - \frac{\kappa}{2} \;\; ; \;\; \lambda = (4\pi G\rho_\circ)^{1/2}.$$

$$\frac{\partial\nu}{\partial t} + \frac{\partial}{\partial x_i}\int v_i f\, d^3v - \frac{\partial\Phi}{\partial x_i}\int f(v_i)]_{-\infty}^{\infty}\, d^2v_{\neq i} = 0$$

$$\sigma_{\rm R}(R) = \sigma_\circ \exp\left(-R/h\right) \;\; ; \;\; M = 2\pi h^2\sigma_\circ$$

$$\Phi(\lambda,\mu,\nu) = -\frac{F(\lambda)}{(\lambda-\mu)(\lambda-\nu)} - \frac{F(\mu)}{(\mu-\nu)(\mu-\lambda)} - \frac{F(\nu)}{(\nu-\lambda)(\nu-\mu)}$$

$$\frac{d^2}{dt^2}\left(\tfrac{1}{2}I_{jk}\right) = \int x_k\frac{\partial}{\partial t}(\rho\bar{v}_j) \;\; ; \;\; K_{ij} = \int \rho\bar{v}_i.\bar{v}_j d^3x + \tfrac{1}{2}\int \rho\sigma_{ij}^2 d^3x = T_{ij} + \tfrac{1}{2}\Pi_{ij}$$

$$\rho(z) = \frac{\langle V_{\rm z}^2\rangle}{2\pi G z_\circ^2}\,{\rm sech}^2\left(\frac{z}{z_\circ}\right) \;\; ; \;\; \langle V_{\rm z}^2\rangle = \pi G\sigma z_\circ \;\; ; \;\; K_{\rm z} = -2\frac{\langle V_{\rm z}^2\rangle}{z_\circ}\tanh\left(\frac{z}{z_\circ}\right)$$

$$R_\circ = \left(\frac{4\pi G\rho_\circ}{9\langle V^2\rangle}\right)^{-1/2} \;\; ; \;\; \langle V^2\rangle^{1/2} \propto \frac{\rho_\circ M(R_{\rm t})}{f\left(R_{\rm t}/R_\circ\right) g\left(R_{\rm t}/R_\circ\right)}$$

$$V_{\rm circ}(R) = \left[V_{\rm disk}^2(R) + V_{\rm bulge}^2(R) + V_{\rm halo}^2(R)\right]^{1/2}$$

$$-K_{\rm R} = \frac{V_{\rm t}^2}{R} - \langle V_{\rm R}^2\rangle\left[\frac{\partial}{\partial R}(\ln\nu\langle V_{\rm R}^2\rangle) + \frac{1}{R}\left\{1 - \frac{\langle(V_\theta - V_{\rm t})^2\rangle}{\langle V_{\rm R}^2\rangle}\right\}\right] + \langle V_{\rm R}V_{\rm z}\rangle\frac{\partial}{\partial z}(\ln\nu\langle V_{\rm R}V_{\rm z}\rangle)$$

$$V_{\rm rot}^2 - V_{\rm t}^2 = -\langle V_{\rm R}^2\rangle\left\{R\frac{\partial}{\partial R}\ln\nu + R\frac{\partial}{\partial R}\ln\langle V_{\rm R}^2\rangle + \left[1 - \frac{B}{B-A}\right]\right\}.$$

$$\frac{\langle V_\theta^2\rangle}{\langle V_{\rm R}^2\rangle} = \frac{-B}{(A-B)} \;\; ; \;\; Q = \frac{\langle V_{\rm R}^2\rangle^{1/2}\kappa}{3.36G\sigma} \;\; ; \;\; Y = V_{\rm rot}\left(\frac{h}{GM_{\rm disk}}\right)^{1/2} \gtrsim 1.1$$

$$4.74\mu = -\omega_\circ - \tfrac{1}{2}\left(\frac{d\omega}{dR}\right)_{R_\circ} R_\circ - \tfrac{1}{2}R_\circ\left(\frac{d\omega}{dR}\right)_{R_\circ}\cos 2l \equiv B + A\cos 2l$$

$$K_{\rm R}(R,z) = -2\pi G\int_0^\infty\int_0^\infty\int_{-\infty}^\infty ukJ_1(kR)J_0(ku)\rho(u,v)e^{-k|z-v|}dv\,du\,dk$$

$$\frac{\partial^2\Phi}{\partial x^2}+\frac{\partial^2\Phi}{\partial y^2}+\frac{\partial^2\Phi}{\partial z^2}\equiv\nabla^2\Phi=4\pi G\rho(x,y,z)\quad;\quad\frac{\partial K_{\rm R}}{\partial R}+\frac{K_{\rm R}}{R}+\frac{\partial K_{\rm z}}{\partial z}=-4\pi G\rho(R,z)$$

$$V_{\rm rad}=R_\circ\left(\frac{V(R)}{R}-\frac{V_\circ}{R_\circ}\right)\sin l\quad;\quad T=R_\circ\left(\frac{V(R)}{R}-\frac{V_\circ}{R_\circ}\right)\cos l-\frac{r}{R}V(R)$$

$$t_{\rm relax}\sim\left(\frac{R^3}{GM}\right)^{1/2}\frac{N}{8\ln N}\quad;\quad\ddot{R}=-\frac{\partial\Phi_{\rm eff}}{\partial R}\quad;\quad\ddot{z}=-\frac{\partial\Phi_{\rm eff}}{\partial z}$$

$$A=\frac{1}{2}\left[\frac{V_\circ}{R_\circ}-\left(\frac{dV}{dR}\right)_{R_\circ}\right]\quad;\quad B=-\frac{1}{2}\left[\frac{V_\circ}{R_\circ}+\left(\frac{dV}{dR}\right)_{R_\circ}\right]$$

$$\frac{x^2}{\tau+\alpha}+\frac{y^2}{\tau+\beta}+\frac{z^2}{\tau+\gamma}=1$$

$$L\propto V_{\rm max}^4\quad;\quad L\propto\langle V^2\rangle^2\quad;\quad R\propto\sigma^{1.4\pm0.15}I^{-0.9\pm0.1}$$

$$V_{\rm R}\frac{\partial f}{\partial R}+V_{\rm z}\frac{\partial f}{\partial z}-\left(\frac{\partial\Phi}{\partial R}-\frac{V_\theta^2}{R}\right)\frac{\partial f}{\partial V_{\rm R}}-\frac{V_{\rm R}V_\theta}{R}\frac{\partial f}{\partial V_\theta}-\frac{\partial\Phi}{\partial z}\frac{\partial f}{\partial V_{\rm z}}=0$$

$$\Phi(R,z)=-2\pi G\int_0^\infty\int_0^\infty\int_{-\infty}^\infty uJ_0(kR)J_0(ku)\rho(u,v)e^{-k|z-v|}\,dv\,du\,dk$$

$$V_{\rm rot}^2(R)=\pi Gh\sigma_\circ\left(\frac{R}{h}\right)^2[I_\circ K_0-I_1K_1]$$

$$u\frac{\partial f}{\partial x}+v\frac{\partial f}{\partial y}+w\frac{\partial f}{\partial z}-\frac{\partial\Phi}{\partial x}\frac{\partial f}{\partial u}-\frac{\partial\Phi}{\partial y}\frac{\partial f}{\partial v}-\frac{\partial\Phi}{\partial z}\frac{\partial f}{\partial w}=0$$

$$r_{\rm tidal}\sim R\left(\frac{m}{3M}\right)^{1/3}\quad;\quad\rho(R)=\frac{\langle V^2\rangle}{2\pi G}R^{-2}\ \ \text{(large R)}$$

$$\vec{g}\left(\frac{g}{a_\circ}\right)\left[1+\left(\frac{g}{a_\circ}\right)^2\right]^{-1/2}=\vec{g}_{\rm N}$$

$$W_{jk}=-\int\rho x_j\frac{\partial\Phi}{\partial x_k}d^3x\quad;\quad\tfrac{1}{2}\frac{d^2}{dt^2}I_{ij}=2T_{ij}+\Pi_{ij}+W_{ij}$$

$$\frac{\partial}{\partial z}(\nu\langle V_{\rm z}^2\rangle)+\frac{\nu\langle V_{\rm R}V_{\rm z}\rangle}{R}+\frac{\partial}{\partial R}(\nu\langle V_{\rm R}V_{\rm z}\rangle)=\nu K_{\rm z}$$

$$V_{\rm rad}=-\tfrac{1}{2}R_\circ\left(\frac{d\omega}{dR}\right)_{R_\circ}r\sin 2l\equiv Ar\sin 2l$$

$$\tilde{\rho}(k,z) = \int_0^\infty u J_0(ku)\rho(u,z)du \quad \rho(u,z) = \int_0^\infty k J_0(kR)\tilde{\rho}(k,z)dk$$

$$\Phi_{\rm eff} = \Phi(R,z) + \frac{L_{\rm z}^2}{2R^2} \quad ; \quad E = \tfrac{1}{2}\dot{R}^2 + \tfrac{1}{2}\dot{z}^2 + \Phi_{\rm eff}(R,z)$$

$$\frac{\partial^2\Phi}{\partial R^2} + \frac{1}{R}\frac{\partial\Phi}{\partial R} + \frac{\partial^2\Phi}{\partial z^2} = 4\pi G\rho(R,z)$$